Engineering Notes

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Estimation of Mass Diffusion Reduction by Drag-Reducing Polymeric Additives

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Nomenclature

c = solution concentration
c^+ = dimensionless concentration, cV_*/j
c_b^+ = dimensionless bulk concentration [Eq. (8)]
D = molecular diffusivity
f = Darcy-Weissbach friction coefficient
j = mass flux
L = characteristic length
N_{Re} = Reynolds number
N_{Sc} = Schmidt number
N_{Sh} = Sherwood number
N_{St} = Stanton number
R = pipe radius
R^+ = RV_*/ν
u = local mean axial velocity
u^+ = u/V_*
u_b = average velocity
V_* = shear velocity
V_{*cr} = critical shear velocity
y = distance from the wall
y^+ = yV_*/ν
α = parameter
δ_1 = edge of diffusion sublayer
δ_1^+ = $\delta_1 V_*/\nu$
δ_2 = edge of viscous sublayer
δ_2^+ = $\delta_2 V_*/\nu$
ν = kinematic velocity
τ = relaxation time
ϕ = parameter of the increase of the diffusion sublayer, dimensionless

THIS Note concerns the possibility of changes in mass diffusion which may be affected by drag-reducing polymeric additives. Such changes may be important when drag-reducing polymeric coating is used.

Meyer[1] assumed that the polymeric additives increase the effective thickness of the viscous boundary sublayer. In a smooth pipe this increase can be calculated from the equation

$$\delta_2^+ - 5.75 \log_{10}\delta_2^+ = 5.5 + \alpha \log_{10}(V_*/V_{*cr}) \qquad (1)$$

where V_{*cr} is a critical value of V_* above which friction loss reduction may be attained.

Assuming that V_{*cr} depends on the viscous and elastic properties of the liquid according to

$$V_{*cr} = (\nu/\tau)^{1/2} \qquad (2)$$

Elata's model[2] is obtained. Assuming that V_{*cr} depends on the viscosity of the solution and a characteristic length of the polymeric molecule according to

$$V_{*cr} = \nu/L \qquad (3)$$

the length scale model is obtained, which is consistent with the approach of Virk[3] as well as that of Van Driest.[4] It is also possible to refer to the model of Seyer and Metzner,[5] who extended Elata's model. Poreh and Paz[6] extended Elata's model for heat-transfer problems by using von Kármán's analogy[7] between momentum and heat transfer in turbulent flow. However, the models (2) and (3) cannot compete with all the experimental data. The possible effect of polymeric additives on mass diffusion processes is investigated by referring here to model (1), although this model is not always applicable, in spite of the fact that the definition of V_{*cr} is general.

According to Levich,[8] the relationship between the diffusion and the viscous sublayers in a smooth turbulent pipe flow is

$$\delta_1^+ = \delta_2^{+0.75}/N_{Sc}^{0.25} \qquad (4)$$

The concentration profiles in the diffusion, viscous sublayers, and the turbulent region are, respectively,

$$c^+ = N_{Sc}y^+ \qquad (5)$$

$$c^+ = N_{Sc}\delta_1^+ + \delta_2^{+3}(1/\delta_1^{+3} - 1/y^{+3})/3 \qquad (6)$$

$$c^+ = N_{Sc}\delta_1^+ + \frac{\delta_2^{+3} - \delta_1^{+3}}{3\delta_1^{+3}} + 5.75 \log_{10}\frac{y^+}{\delta_2^+} \qquad (7)$$

The bulk dimensionless concentration is defined according to

$$c_b^+ = \int_0^{R^+} c^+u^+(R^+ - y^+)dy^+ \Big/ \int_0^{R^+} u^+(R^+ - y^+)dy^+ \qquad (8)$$

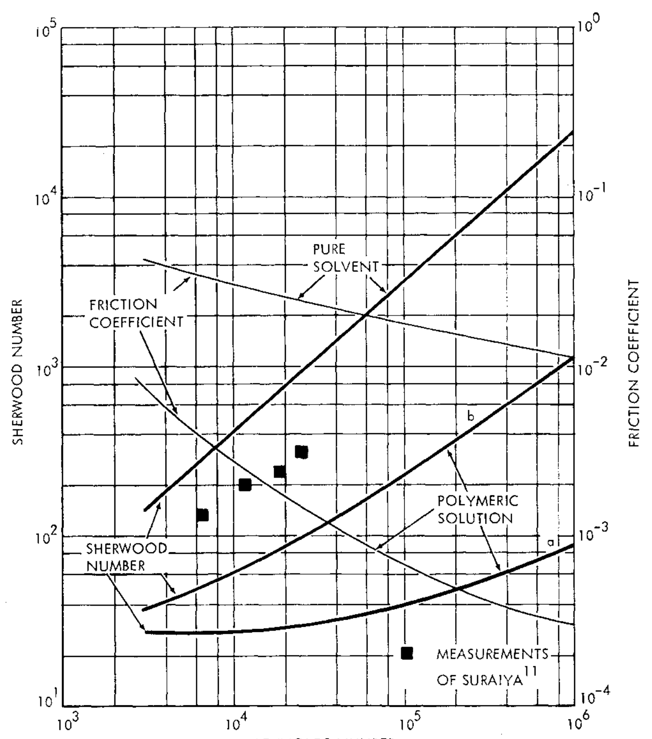

Fig. 1 The effect of polymeric additives on friction coefficient and Sherwood number, when N_{Sc} = 1000, 2R = 0.116 in.

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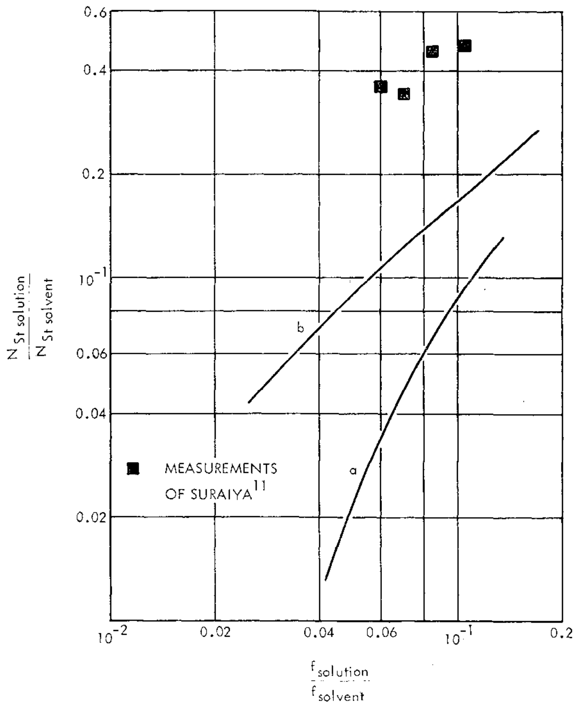

Fig. 2 Changes in the friction coefficient and Stanton number by polymeric additives when N_{Sc} = 1000, 2R = 0.116 in.

If the pipe diameter is large enough, it is possible to neglect the diffusion and viscous sublayers in Eq. (8); therefore, the following is obtained:

$$c_b^+ = N_{Sc}\delta^+ + \frac{\delta_2^{+3} - \delta_1^{+3}}{3\delta_1^{+3}} + \frac{8}{f}^{1/2} + \frac{125}{16}\left(\frac{f}{8}\right) \quad (9)$$

where

$$\left(\frac{8}{f}\right)^{1/2} = 5.75 \log \frac{N_{Re}(f/32)^{1/2}}{\delta_2^+} + \delta_2^+ - 3.75 \quad (10)$$

Sherwood number, which is the mass transfer coefficient,

$$N_{Sh} = 2N_{Sc}R^+/c_b^+ \quad (11)$$

may be found after c_b^+ is calculated according to Eq. (9). The Stanton Number may also be referred,

$$N_{St} = N_{Sh}/(N_{Sc}N_{Re}) \quad (12)$$

By the addition of drag-reducing polymers to the pure solvent, two possible extreme situations may be obtained: a) the diffusion sublayer thickness is not affected by the polymeric additives, and b) the relationship between δ_1^+ and δ_2^+ is not affected by the polymeric additives, i.e., Eq. (4) applies to the flow of the pure solvent as well as to the flow of the polymeric solutions.

As a result of the preceding conditions, we may assume that

$$\delta_1^+ = \frac{\phi\delta_2^{+0.75} + 11.6^{0.75}}{(1 + \phi)N_{Sc}^{0.25}} \quad (13)$$

where ϕ is a dimensionless function. This function should depend upon $\alpha \log_{10}(V_*/V_{*cr})$. The two extreme values of $0 \leq \phi \leq \infty$ express the two extreme situations described before. Wells[9] analyzed the changes in mass diffusion caused by polymeric additives in the case of turbulent flow over a flat plate. His analysis is based upon the empirical investigation of Friend and Metzner.[10] However, the approach suggested here may be applied to the case of the flow over a flat plate as well.

Suraiya[11] measured changes in Sherwood number by the addition of polyethylene oxide WSR-301 to water flowing in a 0.116 in. pipe. His experimental data concerning solutions of 30 ppm are shown in Fig. 1 (in these experiments Schmidt number was about 1000). According to Wells, the critical shear velocity and α for 30 ppm WSR-301 solution are 0.068 fps and 23.0, respectively. On the basis of these data we calculated changes in Sherwood number and friction coefficient by the addition of 30 ppm WSR-301 to water flowing in an 0.116 in. pipe when a Schmidt number is 1000. The results are shown in Fig. 1. The lines designated by a and b show the two extreme cases discussed before.

It is reasonable that polymeric additives reduce Sherwood number more than they reduce the friction coefficient as mass diffusion is extremely dependent upon the flow conditions at the conduit wall. Therefore, from the lines and data shown in Fig. 2, it seems that changes in mass diffusion by polymeric additives are in the range between the two extreme cases described before and that the data of Suraiya[11] is not entirely accurate degradation or maximum drag reduction.

Changes in velocity and concentration profiles by the addition of 30 ppm WSR-301 to flowing water in a 0.116 in. pipe at Reynolds number of 8×10^3 whereas $\phi = 0.8$ are shown in Fig. 3.

References

[1] Meyer, W. A., "A Correlation of the Frictional Characteristics for Turbulent Flow of Dilute Non-Newtonian Fluids in Pipes," *AIChE Journal*, Vol. 12, 1966, pp. 522-525.

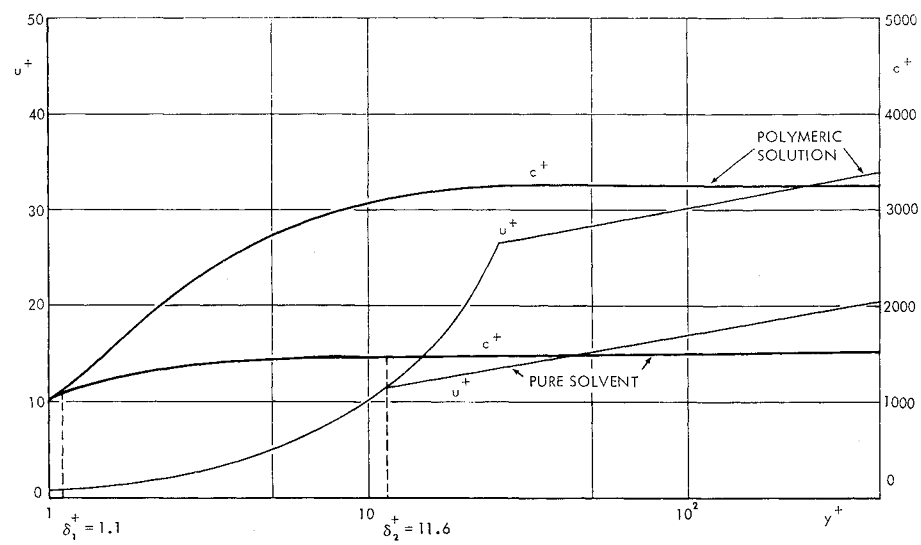

Fig. 3 Changes in velocity and concentration profiles by polymeric additives, when N_{Sc} = 1000, 2R = 0.116 in., N_{Re} = 8 × 10^3, and ϕ = 0.8.